Logarithmic Corrections to Black Hole Entropy v. 2.0

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Logarithmic Corrections

- The leading corrections to the area law for black hole entropy are logarithmic

$$S = \frac{A}{4G} + \frac{1}{2} D_0 \log A + \dots \ .$$

- These corrections can be computed from the low energy theory: only massless fields contribute.

- In some situations the corrections give non-trivial support for a known microscopic description.

- In other situations they offer clues to the nature of the unknown microscopic theory.

Updates in v. 2.0

In principle: computations are straightforward applications of techniques from the 70's.

In recent years, Sen (and collaborators) did what we do, and more.

In practice: computations are cumbersome and intransparent.

Updates in v 2.0 focus on short-cuts that add clarity:

- Interactions with background gravity and graviphoton: employ AdS/CFT, specifically organize fluctuations as ***chiral primaries***.
- Contributions from ***on-shell states only*** (no ghosts).
- Remnant of unphysical states: ***simple boundary states*** .

Collaborators: C. Keeler and P. Lisbao

Setting

- Consider matter in a general theory with $\mathcal{N} \geq 2$ SUSY.

- In terms of $\mathcal{N} = 2$ fields: one SUGRA multiplet, $\mathcal{N} - 2$ (massive) gravitini, n_V vector multiplets, n_H hyper multiplets.

- Setting: focus on extremal black holes $\rightarrow$ it is sufficient to consider the AdS$_2 \times S^2$ near horizon region.

- The final result:

$$\delta S = \frac{1}{12}\left[23 - 11(\mathcal{N} - 2) - n_V + n_H\right] \log A_H \ .$$

- Example (relevant for microscopics): no correction in $\mathcal{N} = 4$ theory with an arbitrary number of $\mathcal{N} = 4$ matter multiplets.

Prelude: Chiral Primaries

- Massless fields in AdS$_2 \times S^2$ organize themselves in short representations of the $SU(2|1,1)$ supergroup.

- CFT language: consider chiral multiplets where (h,j) are

$$(k,k) \otimes 2(k+\frac{1}{2}, k-\frac{1}{2}) \otimes (k+1, k-1)\ .$$

  Possible values of $k = \frac{1}{2}, 1, \frac{3}{2}, \ldots$. ($k = \frac{1}{2}$ extra short).

- The actual values are determined by symmetry.

- Alternatively, they can be computed directly by diagonalization of the field equations.

Spherical Harmonics

- Expansion on S^2 of a single field component with helicity λ: angular momenta $j = |\lambda|, |\lambda| + 1, \ldots$.
- Example: for a gauge field ***all*** components organize themselves into two towers with $j = 1, 2, \ldots$ and two towers with $j = 0, 1, \ldots$
- The ***physical*** components of the vector field components organize themselves into two towers with $j = 1, 2, \ldots$.
- So: the set of physical angular momenta in each $\mathcal{N} = 2$ multiplet is unambiguous.
- Example: the $\mathcal{N} = 2$ vector multiplet has one vector field and two real scalars so the ***physical*** boson towers are: two with $j = 1, 2, \ldots$ and two with $j = 0, 1, \ldots$.
- Mixing is allowed (for same j) but assembly of towers into chiral multiplets uniquely determine conformal weights.

The Spectrum of Chiral Primaries

- Result: the spectrum of (h,j) for all chiral primaries:

$$\begin{aligned} \text{Supergravity}: &\quad 2[(k+2,k+2)\,,2(k+\tfrac{5}{2},k+\tfrac{3}{2})\,,(k+3,k+1)] \\ \text{Gravitino}: &\quad 2[(k+\tfrac{3}{2},k+\tfrac{3}{2})\,,2(k+2,k+1)\,,(k+\tfrac{5}{2},k+\tfrac{1}{2})] \\ \text{Vector}: &\quad 2[(k+1,k+1)\,,2(k+\tfrac{3}{2},k+\tfrac{1}{2})\,,(k+2,k)] \\ \text{Hyper}: &\quad 2[(k+\tfrac{1}{2},k+\tfrac{1}{2})\,,2(k+1,k)\,,(k+\tfrac{3}{2},k-\tfrac{1}{2})] \end{aligned}$$

  Each tower has $k = 0, 1, \ldots$.

- Previous work identified ***one*** more bulk mode in the SUGRA multiplet

$$(1,1)\,,2(\frac{3}{2},\frac{1}{2}),(2,0)\,.$$

- Our result: this field exists ***only*** as a boundary mode.

Example: Constraints for Gravity

- The graviton in D dimensions has $D(D+1)/2$ components, D gauge symmetries (from diffeomorphisms), D constraints (eom's left after gauge fixing).
- So: a graviton has $D(D-3)/2$ physical components.
- In 2D a graviton has -1 degrees of freedom so ***a graviton and a scalar combined has no degrees of freedom***.
- Details: after gauge fixing some "equations of motion" are in fact constraints (there are no time derivatives).
- Exception: the constraint is solved by one specific spatial profile (the zero-mode on AdS_2) so one boundary degree of freedom can be freely specified.
- These ***boundary modes are physical*** (standard in AdS/CFT).

Quantum Fluctuations: Strategy

- All contributions from quadratic fluctuations around the classical geometry take the schematic form

$$e^{-W} = \int \mathcal{D}\phi \, e^{-\phi\Lambda\phi} = \frac{1}{\sqrt{\det\Lambda}} \, .$$

- The quantum corrections are encoded in the heat kernel

$$D(s) = \mathrm{Tr}\, e^{-s\Lambda} = \sum_i e^{-s\lambda_i} \, .$$

- The effective action becomes

$$W = -\frac{1}{2}\int_{\epsilon^2}^{\infty} \frac{ds}{s} D(s) = -\frac{1}{2}\int_{\epsilon^2}^{\infty} \frac{ds}{s} \int d^D x K(s) \, .$$

- We need the constant D_0 (or K_0): essentially the 2nd Seeley-deWitt coefficient aka the trace anomaly.

Simple Heat Kernels in 2D

- The heat kernel for a scalar field on S^2 is ***elementary***:

$$K_S^s(s) = \frac{1}{4\pi a^2} \sum_{k=0}^{\infty} e^{-sk(k+1)}(2k+1) = \frac{1}{4\pi a^2 s}\left(1 + \frac{1}{3}s + \frac{1}{15}s^2 + \ldots\right)$$

- A massless scalar field on AdS$_2$ involves a continuous spectrum:

$$K_A^s(s) = \frac{1}{2\pi a^2} \int_0^{\infty} e^{-(p^2+\frac{1}{4})s} p \tanh \pi p \, dp \, .$$

- The local terms in the AdS$_2$ heat kernel is identical to S^2 except for the sign of the curvature:

$$K_A^s(s) = \frac{1}{4\pi a^2 s}\left(1 - \frac{1}{3}s + \frac{1}{15}s^2 + \ldots\right) \, .$$

Simple Heat Kernels on AdS$_2 \times S^2$

- For a product space heat kernels multiply so for a scalar on AdS$_2 \times S^2$:

$$K_4^s(s) = K_S^s(s) K_A^s(s) = \frac{1}{16\pi^2 a^4 s^2}\left(1 + \frac{1}{45}s^2 + \ldots\right) .$$

- For a Dirac fermion on AdS$_2 \times S^2$:

$$K_4^f(s) = 4K_S^f(s) K_A^f(s) = -\frac{1}{4\pi^2 a^4 s^2}\left(1 - \frac{11}{180}s^2 + \ldots\right) .$$

- A benchmark for results in $\mathcal{N} = 2$ theory: a "free hyper"

$$K_4^{\min}(s) = 4K_4^s(s) + K_4^f(s) = \frac{1}{4\pi^2 a^4 s^2} \cdot \frac{1}{12}s^2 .$$

- The leading $1/s^2$ singularity cancels: no cosmological constant for equal number of fermion and bosons.

- The $1/s$ order also cancels: this is an accident.

The AdS_2 Perspective

- The canonical heat kernel on AdS_2: a massless field $h = 1$.

- A field with conformal weight h (mass $m^2 = h(h-1)$) and $SU(2)$ quantum number j (degeneracy $2j+1$):

$$K_A(h, j; s) = K_A(h = 1, j = 0; s)\, e^{-h(h-1)s}(2j+1)\,.$$

- A free $4D$ boson is a tower of $2D$ bosons with $(h, j) = (k+1, k)$ with $k = 0, 1, \ldots$ so

$$\begin{aligned} K_4^s(s) &= K_A^s(s) \cdot \frac{1}{4\pi a^2} \sum_{k=0}^{\infty} e^{-sk(k+1)}(2k+1) \\ &= \frac{1}{16\pi^2 a^4 s^2}\left(1 + \frac{1}{45}s^2 + \ldots\right)\,. \end{aligned}$$

- The sum over the tower of AdS_2 fields computes the factor from the heat kernel on S^2.

Example: Bulk Vector-Multiplet

- The conformal weights for fields in supergravity are "shifted" from the free values.
- The fermions in the vector multiplet are canonical but bosons interact: this is the ***attractor mechanism***.
- The "shifted" sum on S^2 for all four physical bosons:

$$K_4^{V,b}(s) = \frac{2K_A^s(s)}{4\pi a^2}\sum_{k=0}^{\infty}\left(e^{-sk(k+1)}(2k+3)+e^{-s(k+1)(k+2)}(2k+1)\right)$$

$$= \frac{1}{4\pi^2 a^4 s^2}\left(1+\frac{1}{45}s^2+\ldots+\frac{1}{2}s(1-\frac{1}{3}s)+\ldots\right) .$$

- Heat kernel for the full vector multiplet including fermions:

$$K_4^V(s) = \frac{1}{4\pi^2 a^4}\left(\frac{1}{2s}-\frac{1}{12}+\ldots\right) .$$

- A 1/s term was generated by interactions.
- The constant term changed sign due to interactions.

The Vector-Multiplet: Boundary

- Gauge invariance of the vector field requires special considerations.
- Two components of the vector cancel: unphysical states (violate gauge condition) and physical (but pure gauge).
- The boundary state: one of the would-be gauge functions is not normalizable so ***one*** state survives.
- Alternatively: one equation of motion is a ***constraint*** so one spatial profile survives.
- The ***boundary state is a massless boson on*** S^2:

$$-\nabla^I \delta \mathcal{A}_I = -\nabla^2 \Lambda = 0$$

- Final result for the heat kernel (bulk+boundary, bosons+fermions):

$$K_4^V(s) = \frac{1}{4\pi^2 a^4}\left(\frac{1}{2s} - \frac{1}{12}\right) + \frac{1}{4\pi^2 a^4}\left(\frac{1}{2s} + \frac{1}{6}\right) = \frac{1}{4\pi^2 a^4}\left(\frac{1}{s} + \frac{1}{12}\right) .$$

The Graviton Multiplet

- ***Bulk modes***: physical bosons and fermions all have conformal weight shifted from the free value. Details are determined by symmetry.

- ***Bosonic boundary modes***: 4 from $4D$ diffeomorphisms and 1 from gauge symmetry $\rightarrow 4 + 1 = 5$ towers of boundary modes.

- ***Fermionic boundary modes***: 2 preserved SUSY's $\rightarrow 4$ towers of boundary modes.

- Boundary modes are related to gauge symmetries with spectrum determined by their equation of motion (the gauge conditions).

- Boundary modes for diffeomorphisms ***acquire a mass***

$$(g_{IJ}\nabla^2 + R_{IJ})\xi^J = 0\ .$$

and they also mix with gauge fields.

- These towers are related to the pure gauge modes in bulk (rather then physical bulk modes) so details are not fixed by symmetries.

- For each field we sum over the partial wave tower with masses (conformal weights) shifted due to interactions.

- The full heat kernel (bulk+boundary, bosons+fermions):

$$K^{\text{grav}} = \frac{1}{4\pi^2 a^4}\left((\frac{1}{2s} - \frac{1}{12}) + (\frac{1}{2s} - \frac{5}{6}) \right) = \frac{1}{4\pi^2 a^4}\left(\frac{1}{s} - \frac{11}{12} \right)$$

Explicit Computations: Bosons

- The bosonic theory is just canonical Einstein gravity coupled to Maxwell.

- There are $10 + 4 = 14$ field components.

- We ***dualize*** all 2D bosons to scalars in AdS$_2$ and then diagonalise the resulting action.

- This is messy: the matrix of kinetic terms and the matrix of mass-terms ***do not commute*** $\to$ cannot be simultaneously diagonalised $\to$ generalised eigenvalues and generalized eigen-vectors.

- Ultimately: 14 towers of 2D fields divided into: 5 (projected out by the ***gauge condition***), 5 (***pure gauge***), and 4 ***physical*** towers.

Explicit Computations: Results

- The ***physical*** fields are ***true eigenvectors*** with eigenvalues (masses) corresponding to the conformal weights already determined by symmetry.

- The 5 ***pairs*** of ***unphysical*** towers have masses $m^2 = l(l+1) \pm 2, m^2 = l(l+1)$ that generally give ***irrational conformal weight***.

- The ***boundary modes*** are formally pure gauge (but with non-normalisable gauge function).

- Boundary modes ***mix***: the gauge transformation ($l \geq 0$) has an admixture of S^2 diff's (for $l \geq 1$); the S^2 diff's for $l \geq 1$ have a compensating gauge transformation.

- Mixing $\rightarrow$ physical boundary modes have ***rational*** conformal weight: $m^2 = l(l-1), (l+1)(l+2) \rightarrow h = l, l+2$.

The Harmonic Condition

- Boundary modes are related to an ***ambiguity*** when dualizing higher spin fields to scalars, eg.:

$$A_\mu = \nabla_\mu A_\| + \epsilon_{\mu\nu}\nabla^\nu A_\perp$$

- ***Harmonic modes*** $\nabla^2 A_0 = 0$ can be written in both forms. They should not be overcounted; and these are boundary modes.

- We dualize ***gravity*** as

$$H_{\{\mu\nu\}} = \nabla_{\{\mu}\nabla_{\nu\}}H_\times + \nabla_{\{\mu}\epsilon_{\nu\}\lambda}\nabla^\lambda H_+$$

- "Harmonic" modes $\nabla^2(\nabla^2 - 2)H_0 = 0$ can be written in both forms; only the $m^2 = 2$ components are boundary modes.

- Some AdS_2 diff's are CKVs:

$$\nabla_\mu\xi_\nu + \nabla_\nu\xi_\mu - g_{\mu\nu}\nabla^\lambda\xi_\lambda = 0$$

  The ***boundary modes*** are precisely those that are ***not CKVs*** but satisfy the "harmonic" condition $(\nabla^2 - 1)\xi_\mu = 0$.

- These $H_{\{\mu\nu\}}$ are ***quadratic holomorphic differentials*** on AdS_2.

The Quadratic Divergence

- After sum over ***bulk and boundary modes***:

$$K_{\text{phys}} = \frac{1}{4\pi^2 a^4}\left[(\frac{1}{s} - \frac{11}{12}) + (\mathcal{N} - 2)\cdot(-\frac{1}{s} + \frac{11}{12}) + n_V(\frac{1}{s} + \frac{1}{12}) + n_H(-\frac{1}{s} - \frac{1}{12})\right]$$

- The net result for the quadratic divergence (the 1/s term): ***alternating sign***.

- This term is from ***interactions*** in bulk and ***counting*** of boundary modes.

- Special case $\mathcal{N} \geq 4$ theory (with any matter): ***quadratic divergence cancels*** (a consistency check).

- For $\mathcal{N} = 3$: ***all divergences cancel*** for any $n_V = n_H$.

- For $\mathcal{N} = 2$: quadratic divergence $\propto 1 + n_V - n_H$.

4D Zero Modes

- 4D zero modes: AdS_2 ***boundary modes and also massless*** on S^2.
- Physical origin: the ***global part*** of each unbroken gauge symmetry.
- Zero-modes play a special role in the 4D heat kernel:

$$D(s) = \sum_i e^{-s\lambda_i} = \sum_{\lambda_i \neq 0} e^{-s\lambda_i} + N_0$$

- The path integral reduces to an ***ordinary*** integral

$$e^{-W} = \int \mathcal{D}\phi_0 = \mathrm{Vol}[\phi_0] \sim \epsilon^{-N_0\Delta} \ .$$

- Correction due to all 0-modes

$$K_{zm} = \frac{1}{8\pi^2 a^4} \cdot \left[6 \cdot (2-1) - 8 \cdot (\frac{3}{2} - \frac{1}{2}) \right] = \frac{1}{4\pi^2 a^4} (-1) \ .$$

- Note: ***much of the literature accounts incorrectly for 0-modes.***

The Quadratic Divergence

- After sum over ***all*** modes

$$K_{\rm phys} = \frac{1}{4\pi^2 a^4}\left[(\frac{1}{s} - \frac{23}{12}) + (\mathcal{N}-2)\cdot(-\frac{1}{s}+\frac{11}{12}) + n_V(\frac{1}{s}+\frac{1}{12}) + n_H(-\frac{1}{s}-\frac{1}{12})\right]$$

- The R^2 correction (the constant term) in $\mathcal{N}=3$ SUGRA is computed by zero-modes.

- Logarithmic corrections to the black hole entropy

$$\delta S = \frac{1}{12}\left[23 - 11(\mathcal{N}-2) - n_V + n_H\right]\log A_H \ .$$

Example: Reissner-Nordström

The minimal bosonic theory: gravity+Maxwell.

Contributions are the bosonic terms from the $\mathcal{N} = 2$ SUGRA multiplet:

- Four free bulk bosons (2 gravity + 2 gauge field): $\delta S = -\frac{1}{45} \log A_H$.
- Interactions (bulk bosons not quite free): $\delta S = -\frac{3}{2} \log A_H$.
- 5 Boundary modes (4 gravity+1 gauge field): $\delta S = -\frac{5}{6} \log A_H$.
- Zero-modes: $\delta S = -3 \log A_H$.

Total: $\delta S = -\frac{241}{45} \log A_H$.

(Fermions in SUGRA multiplet add $\delta S = \frac{1309}{180} \log A_H$)

Summary

We re-evaluated quadratic fluctuation determinants around an $\text{AdS}_2 \times S^2$ near horizon geometry.

Some features of our strategy:

- Focus on states that are on-shell.
- Interactions due to background: encoded in chiral primaries.
- Compute also the renormalization of the gravitational coupling constant (quadratic divergence, 1/s term in the heat kernel).
- Contributions from bulk (4D), Boundary (2D), and Zero-mode (0D).
- Explicit decoupling of equation of motion $\rightarrow$ expressions for all modes including boundary modes.